Jonathan McDowell 1982 May 6

PART III ESSAY

THE MISSING MASS

by Jonathan McDowell

"Apparent velocity differences exist in the Coma cluster of at least 1500-2000 km/s ... If we assume that the Coma system has reached a dynamical steady state, it follows from the virial theorem that $\bar{\epsilon}_k = -\frac{1}{2}\bar{\epsilon}_p$"

(mean kinetic energy = -1/2 mean potential energy).

(Estimating that for 800 galaxies of $10^9 M_\odot$ each, the r.m.s. velocity should be 80 km/s,)

"... the mean density must be 400 times bigger than that derived on the grounds of luminous material ... If this should be verified, the astonishing result would therefore follow that dark material is present in much larger quantity than luminous material."

(translated from FRITZ ZWICKY, 'Die Rotverschiebung von extragalaktischen Nebeln' Helvetica Physica Acta 6, p.124-125 (1933))

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I declare that this essay is my own unaided work except insofar as it uses the quoted references and except as acknowledged below.

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1. Introduction

1.1 The Case of the Missing Mass

> "When you have eliminated the impossible, whatever remains, however improbable, must be the truth."
>
> Sherlock Holmes ('The Sign of Four')
>
> -but how can you be sure you've covered all the possibilities?

Fifty years ago Zwicky (1933) discovered evidence for the possibility that much of the mass of clusters of galaxies lies in unseen material. This problem is far from being solved today: in fact, it has grown to encompass all of modern astronomy from the astrophysics of the local solar neighbourhood to the relation between particle physics and cosmology.

Although the interpretation of the observations is still the subject of much debate, many astronomers now believe that the fundamental discrepancy is well established: masses determined by dynamical means are grossly larger than the estimated masses of the luminous matter making up the system in question, and this discrepancy seems to increase with increasing scale.

Is most of the universe invisible to us? What form is it hidden in? The first part of this essay briefly describes the observations: chapter 2 discusses various methods of determining the mass of a system from its dynamics: chapter 3 reviews the current state of the observations. Faber and Gallagher (1979) (hereafter FG) gave a comprehensive review of the observations and the problems with their interpretation: it would be impossible (and not very useful) to repeat many of the fine points and subtle causes of error they discuss in this essay. Chapter 4 explores the many different ideas for possible culprits: all these suspicious characters have alibis, but some are more shaky than others. Detective work over the past few years seems to clear some, while one light-footed, apparently insignificant citizen of spacetime has recently been implicated in almost every major astronomical crime, from thefts from the Sun's core to plots to dominate, and bring about the very end of, the Universe itself. But first, we must set the scene ...

1.2 Overview

'A spiral galaxy consists of a spheroidal nucleus of stars and a flat disc made of stars, gas and dust with a pattern of spiral arms marking a wave of star formation. It also has a 'halo' of small mass contaning some stars and globular clusters. Its total mass is about 10^{11} solar masses, and it, its brothers, and its elliptical cousins make up our universe.'

This picture is now in deep trouble. It was believed that the density of matter falls off rapidly at the edge of the luminous part of a galaxy, but the evidence to be discussed suggests that the fall-off is only inverse square, and so the total mass within radius r, M(r), is proportional to r. It is not known where this massive halo stops, and it may be that groups and clusters of galaxies contain much material not associated with individual galaxies. The estimated masses of clusters of galaxies are much greater than the sum of the estimated masses of the component galaxies. The fact that the light falls off more rapidly than the mass suggests that the extra matter is darker than ordinary stars.

The most important observational quantity in these studies is the radial velocity , of galaxies, or parts of galaxies, or clusters. In this essay, radial velocity will always mean the recession velocity along the line of sight to the Earth, never a rate of change of radial coordinate of the system in question. This radial velocity is, of course, determined from the Doppler redshift in the spectra of these objects. In the next chapter we see how masses are determined from radial velocity studies.

2. Measuring Mass

2.1 Measuring Mass Dynamically

Consider a spherical self-gravitating system with mass M(R) contained within radius R. A theorem by Newton tells us we may consider the gravitational effect on a test particle in a circular orbit of radius R as equal to that due to a point mass M(R) at the centre. Then from Newton's law of gravitation $\frac{V^2}{R} = \frac{GM(R)}{R^2}$ where v is the velocity of the particle. So,

$$\therefore \quad M(R) = \frac{V^2 R}{G} \qquad (1)$$

so if we know R and V(R) we may determine M(R). The formula (1) is the basis of work on measuring mass by dynamical means.

Unfortunately, as always in astronomy, life is not that simple. In fact, every assumption going into (1) is suspect or blatantly inadequate. However the intuitive feeling that (1) should represent at least some qualitative measure of the mass has led to justifications of the use of forms of (1) in more general contexts which we now consider.

Let a test particle be in orbit in the potential of a collection of masses, with position vector $\underline{r}$ with respect to the centre of the system. Now

$$\frac{d^2}{dt^2}\tfrac{1}{2}\underline{r}\cdot\underline{r} = \underline{r}\cdot\ddot{\underline{r}} + \dot{\underline{r}}\cdot\dot{\underline{r}} = \underline{r}\cdot\ddot{\underline{r}} + v^2 \qquad (2)$$

Let the potential be $\psi = \sum_{i=1}^{N} \frac{G m_i}{|\underline{r}-\underline{r}_i|}$ for discrete masses,

in the continuum limit: $\psi = \int \frac{G\rho(\underline{r}')}{|\underline{r}-\underline{r}'|} d^3\underline{r}'$

Then

$$\rho(\underline{r})\frac{d^2}{dt^2}\tfrac{1}{2}r^2 = \rho(\underline{r})\,\underline{r}\cdot\nabla\psi + v^2\rho(\underline{r})$$

Integrating over all $\underline{r}$ and after some manipulation, $\frac{1}{2}\frac{d^2}{dt^2}I = 2T - \Phi$ (3)

where $I = \int \rho(\underline{r}) r^2 d^3\underline{r}$

$T = \frac{1}{2}\int \rho(\underline{r}) v^2 d^3\underline{r}$, the kinetic energy

$\Phi = \frac{1}{2}\int \rho(\underline{r})\psi(\underline{r}) d^3\underline{r}$, the total potential energy.

In a steady state we expect I to be a constant on average; for a cluster of galaxies, initially I will decrease as the cluster collapses and forms, then it will reach an equilibrium state with constant time-averaged I . The cluster is then said to be virialized. If we assume $\ddot{I}=0$ at the time we observe, then

$$2T = \Phi \qquad (4)$$

For N particles i=1,...N of mass m_i, define:

$$V^2 = \frac{\sum m_i V_i^2}{\sum m_i}$$

$$M = \sum m_i$$

$$R = \left(\sum_i m_i\right)^2 \Big/ \left(\sum_i \sum_j m_i m_j / |\underline{r}_i - \underline{r}_j|\right)$$

Then

$$T = \tfrac{1}{2} M V^2$$

$$\Phi = GM^2/R$$

and so

$$M = V^2 R / G$$

But to calculate v^2, R, we need to know the masses or at least the mass ratios of the particles, as well as their positions and velocities. In practice we only have their radial velocities and their projected separations. We assume the galaxies are of equal mass and spherically distributed. A method by Schwarzschild (1954) involves counting galaxies in strips. Let S(q) be the number of galaxies in a strip at perpendicular distance q from the centre of the cluster it can be shown that

$$\Phi = -2G \int_0^{R1} S^2(q)\, dq$$

where R1 is the 'edge' of the cluster

$$M = 2 \int_0^{R1} S(q)\, dq$$

so let

$$R = \frac{\left(\int S(q)\, dq\right)^2}{\int S^2(q)\, dq} \qquad (5)$$

then

$$M = \frac{V^2 R}{G}$$

Otherwise, we may assume a density distribution in the cluster (e.g. a de Vaucouleurs law) and calculate

$$\Phi = \frac{GM^2}{R'}$$

where R' is a length which may be found from fitting to the observed galaxy counts.

For small groups of galaxies σ^2 will be anomalously large if galaxies which are not group members are accidentally included; this is less of a problem in large clusters but will still lead to errors and makes the virial theorem unreliable due to the need to study the outer regions where contamination is severe. (Rood et al.,1972)

However the worst uncertainty is in the potential energy term. In particular,it has been suggested (Ozernoy and Reinhardt 1979,Wesson 1980) that substructure in clusters of galaxies should be taken into account and that the missing mass problem may be due to the binding energy of binary galaxies. For a cluster with N galaxies of mass m,

$$\Phi \sim \frac{Gm^2N^2}{R}$$

so a binary of radius r will have comparable potential energy if

$$r \sim R/N^2$$

Ozernoy and Reinhardt claim that NGC 4874 and NGC 4889 are a binary whose binding energy represents the missing mass of the Coma cluster. Geller and Beers (1982) report evidence for substructure in clusters of galaxies.

Another possible source of error lies in the assumption that the mass is distributed like the light. If there is a missing mass component quite uncorrelated with thevisible galaxies,estimates of Φ would be incorrect.

We also require $\ddot{I}=0$: if the cluster is not bound,or still contracting, the virial theorem will not apply. This is of concern for small groups,but the large clusters must surely be bound, and if the time a galaxy takes to cross the cluster is much less than the age of the cluster (i.e. $\lesssim 1/H_0$) we may assume that the cluster is at least bound and probably in virial equilibrium. If the cluster is still contracting,we have $\dot{I}<0$ and $|\dot{I}|$ decreasing,so $\ddot{I}>0$.

Hence,

$$M = \frac{V^2R}{G} - \frac{\ddot{I}R}{2GM} < \frac{V^2R}{G}$$

so this would cause masses to be overestimated.

For analysis of the galaxy's rotation curve and of galaxy satellite orbits we use (2):

$$\frac{d}{dt}\frac{1}{2}r^2 = \underset{\sim}{r}\cdot\underset{\sim}{\nabla}\psi(\underset{\sim}{r}) + v^2$$

If r=const, $v^2 = -\underset{\sim}{r}\cdot\underset{\sim}{\nabla}\psi$ so we define the circular velocity by

$$V_c^2 = -\underset{\sim}{r}\cdot\underset{\sim}{\nabla}\psi \qquad (6)$$

so

$$V_c^2 = V^2 - \frac{1}{2}\frac{d^2}{dt^2}r^2$$

If we average over many objects we hope $\overline{\frac{d^2}{dt^2}r^2} = 0$

i.e.

$$V_c^2 = \overline{V}^2$$

and again $\overline{v^2}$ is related to v_r^2 ,the radial velocity.($\overline{V^2} = 3v_r^2$ if the orbit distribution function is isotropic.)

For rotation curves of edge-on spirals we assume that $v_r^2 = v^2$, ie circular motions. If the motion is in fact highly noncircular this will affect our results by a factor of 2 or 3.

Having determined $\overline{v^2}$,and hopefully R ,we may then use (1). It is obvious that there is at least one component of a spiral galaxy- the disc- which lacks spherical symmetry. Now,Lynden-Bell and Pineault (1978) summarize work by Mestel (1963) showing that a flat rotating disc with v=constant out to r=R also satisfies $M(R)=V^2R/G$. However some authors suggest that approximately flat rotation curves can be generated by the composite potential of spheroid,disc,and halo,with the property that M(R)/R decreases at large R,i.e. spherical density falls off faster than $1/R^2$.(e.g. Bachall,Schmidt,and Soneira 1982).Various dynamical arguments that independently suggest spiral galaxies ought to have massive spherical haloes (e.g. Ostriker and Peebles 1973) will not be discussed here due to lack of space.

For clusters and elliptical galaxies whose density distribution can be fitted by an isothermal sphere,the central density is given by the core radius R_c (where the projected number density ,or surface brightness,drop to 1/2 the central value) and the radial velocity dispersion σ^2 :

$$\rho_0 = \frac{9\sigma^2}{4\pi G R_c^2} \qquad (7)$$

and the core mass-to-luminosity ratio is

$$\frac{\rho_0}{\ell_0} = \frac{9\sigma^2}{2\pi G I_0 R_c} \qquad (8)$$

where I_c is the central surface brightness ,and $I_0 R_c$ is less sensitive to observational error than R_c^2. (Rood et al.,1972,Peebles 1980).
(Actually I_0 is 4πx surface brightness; converting from magnitude to luminosity automatically brings in this factor,accounting for the difference in formulae between Rood et al. and others,and Peebles.)

A more recently developed method for studying the distribution of mass on large scales involves analysing deviations from the Hubble flow. 'Cosmic virial theorems' relate the velocity dispersion of galactic peculiar velocities to the mass density by making assumptions about galaxy clustering. Peebles and coworkers have found that galaxies have a spatial covariance function $\xi(r) \propto r^{-1.8}$; this is the measure of how many extra galaxies above the global average are expected at a point known to be r away from some galaxy. Peebles(1980) discusses how to estimate peculiar velocities: Davis et al (1978) give a discussion of methods of estimating Ω ,the cosmological density parameter: high values (Ω>0.1) are obtained.

Some other ways of estimating masses are discussed in the next chapter in the context of the relevant observations.

2.2 Mass-to-Light Ratios

To try to understand the problem of the missing mass,it is conventional to work in terms of M/L,the ratio of the total mass within radius R to total luminosity within radius R . (although 'local' M/L is also used in some cases.) Note that the total M/L for an object is not well defined if the outer boundary of the object is not known. M/L is a useful concept since if a region has a given M/L we know it cannot contain exclusively material of a lower M/L. M/L is qouted in units of solar masses per solar luminosity ($\mathcal{M}_\odot/\mathcal{L}_\odot$).

2.3 Measuring Luminosity

To measure the luminosity of a cluster of galaxies,we must not only sum the light from its visible members but also allow for faint members too. In a nearby group we may see dwarf elliptical galaxies but in a distant cluster we may see only the brightest members.So,we need to know both the distance modulus $|m-M|$ of the cluster and the luminosity function ϕ (M) (the number of galaxies of absolute magnitude M per unit volume),so we may extrapolate to faint magnitudes. Unfortunately galaxies of equal luminosities but different radial light distribution can have very different surface brightnesses so estimating luminosity is tricky; indeed it has been suggested (Disney,1982) that preferential selection of galaxies of particular surface brightness could mask the existence of a large population of galaxies with different luminosity distributions ,lost in the sky background. (However,one might have hoped to see these at 21 cm.)

The luminosity of a galaxy is estimated from its apparent magnitude in some band: B will be used here in general for compatibility with FG,except where explicitly noted. The values further have to be corrected for absorption by intervening gas in our galaxy;and this must also be done for the source galaxy's self-absorption if we wish to compare results with the solar neighbourhood. Converting to B magnitudes from other bands is non-trivial;indeed, Bothun and Schommer (1982) suggest that some values of M/L for hydrogen in spirals are in error due to faulty conversion of Zwicky estimated magnitudes to m_B.

For cosmology it is interesting to express mass density in terms of the density parameter Ω. The luminosity density of the universe has been estimated from a redshift survey by Kirshner, Oemler and Schechter (Peebles 1979) to be $\mathcal{L} = 10^8\ (H_0/50)\ \mathcal{L}_\odot\ \mathrm{Mpc}^{-3}$, where H_0 is Hubble's constant in km/s/Mpc. (This is actually in J band, but Davis and Huchra (1982) get the same number for Zwicky magnitudes.) Then since the critical density is

$$\rho_c = \frac{3H_0^2}{8\pi G} = 6.9\times10^{10}\left(\frac{H_0}{50}\right)^2 \mathcal{M}_\odot\ \mathrm{Mpc}^{-3},$$

we have

$$(\rho/\ell)_{crit} \approx 700\left(\frac{H_0}{50}\right) \mathcal{M}_\odot/\mathcal{L}_\odot$$

so

$$(M/L)_{global} = 700\ \Omega\ \left(\frac{H_0}{50}\right). \qquad (9)$$

Note that this means that if M/L is typically 700 for clusters, then the density is critical universally and not just within the enhanced density of a cluster. (since we assume all the luminosity is from galaxies, and most galaxies are in groups or clusters.)

2.4 Measuring Distance

$$M \sim V^2R \sim 1/H_0$$

$$L \sim 4\pi d^2 \times \text{intrinsic power} \sim 1/H_0^2$$

so,

$$M/L \sim H_0$$

So, in general, M/L scales with Hubble's constant. Hence in this review I will adopt H_0=50 km/s/Mpc, although the recent results of de Vaucouleurs and others indicate that the true value may be 100, despite the claims of Sandage and Tammann. (but that is another essay!) If H_0 is indeed larger, M/L will be larger and the missing mass problem more acute, so in this case adopting a low H_0 is the conservative approach.

2.5 Measuring Mass Hydrodynamically

The discovery of hot X-ray emitting gas in clusters of galaxies has led to a new method of measuring the mass of a cluster's central galaxy. For gas in a spherical potential with mass M(r) within radius r,

$$M(r) \propto T\left(\frac{d\log\rho}{d\log r} + \frac{d\log T}{d\log r}\right)$$

Fabricant et al (1980) estimate $M \sim 10^{13} M_{\odot}$ within 230 kpc of M87 and believe $d\log T/d\log r=0$, but Binney and Cowie(1981) fit the same data to gas confined by the pressure of external cluster gas, with a cooling flow $dT/dr>0$, and a mass of only $5\times10^{11} M_{\odot}$ (They point out that a $M(r) \propto r$ halo would not have a significant effect on the gas in question, so their result does not rule out such a halo.) So although this method may be important in future, it does not at present provide good evidence for or against missing mass.

2.6 Measuring Mass by Counting Photons

In standard hot big bang theory, most of the helium in the universe is produced in the first few minutes. The fraction Y_p of the mass of the universe turned into helium is determined by the neutron-proton ratio at the time when the density is high enough and the temperature low enough for deuterium synthesis. This time is earlier if the baryon-to-photon ratio $\eta = n_{baryons}/n_{photons}$ is higher: then Y_p is also larger.

Now we think we know how many photons there were, for we observe the cosmic microwave background and we believe that the only significant change to the density of photons has been due to electron-positron annihilation which increases photon density n_{γ} by a factor 11/4. We also believe that helium synthesis since the big bang is much less than Y_p and that little helium has been destroyed. Searching for old objects of low helium abundance will enable us to estimate Y_p. Recent values give $Y_p \leq 0.25$ and imply $\eta < 10^{-9}$ or $\Omega < 0.14$ (if

T_{CBR}=2.7K). (Olive et al,1981) This corresponds to

$$(M/L)_{global} < 100 \quad \text{if all mass is in baryons.}$$

If H_0=100,the corresponding limits are $\Omega<0.03$ and $(M/L)_{global} <42$.

Now,if this calculation is correct, then if average values of M/L for clusters are more than 100,at least some of the 'missing mass' was not in the form of baryons at the time of nucleosynthesis.For this reason,discussion of explanations of the missing mass is divided into those involving mass in the form of baryons at the time of nucleosynthesis (section 4.3) and those explanations not involving such mass.(section 4.4) The distinction is unimportant for galactic halo mass except insofar as it seems likely that missing mass in clusters has the same origin (tidally stripped from the haloes ,perhaps?).

3. Mass-to-Light Ratios of Galaxies and Clusters of Galaxies

3.1 Clusters of Galaxies

Since Zwicky's original study of the Coma cluster estimates of its M/L remain high. Abell(1977) derived $M/L_V=120$, having estimated an unusually high luminosity relative to other workers, and taking $v^2=2\sigma^2$, but most recent studies have higher values. FG have restudied earlier data and find a median M/L_V of 290. Dressler (1981) gets $M/L_V \sim 500$ for the cluster Abell 2029. As FG discuss, and as indicated in the last chapter, these values could be overestimates, certainly by a factor of 2 or so ; Peebles(1980) concludes that the assumption that the velocity distribution is isotropic causes only a small error, but the potential and 'spurious member' problems are very important.

Hoffmann et al. (1980) model the Virgo cluster as a radial condensation in a Friedmann model and derive from observed galaxy velocities a value $M/L=325\pm50$ within 6° of the centre. Peebles (1979) analysed the Kirshner, Oemler and Schechter redshift survey to estimate roughly $\Omega\sim0.4\pm0.2$ or $M/L\sim270$ on very large scales. Recent work suggests the Local Group is falling towards Virgo with a velocity of the order of several hundred km/s. (eg Tonry and Davis 1981b). Davis and Huchra (1982) analyse an extensive redshift survey and assuming Virgo infall velocities between 250 and 500 km/s derive $\Omega=0.2$ to 0.5 or $M/L=140$ to 350.

Although contamination may affect M/L derived from the virial theorem, values found using core fitting should not suffer from this problem and yet almost all the results are of the same order, $M/L_V>100$. (M/L_B is somewhat higher.) We next investigate the masses of individual galaxies, revealing the problem of the missing mass.

3.2 Spiral Galaxies

Observations of radial velocity v(R) have been made out to large distances from the centres of spirals in the optical and in the radio (neutral hydrogen). In a recent detailed study of optical rotation curves of 21 Sc galaxies, (Rubin et al 1980, Burstein et al 1982) all were found to have rotation curves which were flat or still rising at the edge of the optical galaxy. Curves of small galaxies are still rising at their isophotal radius (essentially, the faintest visible extent) while those of large galaxies rise abruptly and then remain flat to large distances: the largest galaxy in the sample, UGC 2885, has a flat rotation curve out to at least 25 kpc and possibly 120 kpc. Burstein et al show that a single form of the mass distribution with one scaling parameter can fit almost all the data.

HI rotation curves also remain flat (Bosma 1978 thesis, quoted in FG). FG give M/L values calculated for galaxies out to the Holmberg radius. (this is a radius defined by an apparent surface brightness contour and so does not correspond to any physical scale in the galaxy.) These range from about 2 for Magellanic type irregulars to 6-10 for Sa and S0 galaxies. However, Krumm and Salpeter (1979) get typical values of 4-5 (converting to H_o=50) from HI observations of 14 spirals out to the Holmberg radius, finding little correlation with Hubble type. Bosma (1981) finds $M/L \sim 7$. Most authors deduce from the flat rotation curves that $M(R) \propto R$ in the outer parts of spiral galaxies so M/L increases drastically. (Bachall and Soneira 1980 review the argument for a massive halo.)

3.3 Elliptical galaxies

M/L can be estimated from velocity dispersions σ in ellipticals using the virial theorem but as FG note , σ is usually only known in the nucleus, and the potential energy cannot be reliably evaluated since even the luminosity distribution in these galaxies is poorly known, never mind the mass distribution. Fitting core velocity dispersions to eq. 8 is likely to give more realistic results. Having said this, globally calculated values by Tonry and Davis (1981a) have a similar scatter to core M/L values determined by Schechter (1980). The Tonry and Davis values are centered on 14, while Schechter's values range from 2 to 23 with a mean (in the log) of 8.5 . Schechter shows that corrections for the nonsphericity of the galaxy are unimportant. Malumuth and Kirshner (1981) use King's method (ie equation 8) to get core values of M/L = 6.5 ± 0.7 for ellipticals and 9.9 ± 0.8 for the brightest galaxies in clusters.

To summarize,mass-to-light ratios appear systematically higher for ellipticals relative to spirals. This is understandable as they lack the young,hot,bright stars found in spiral arms. However the M/L ratios are a factor of 10 or more lower than for clusters of galaxies. This is the problem of the missing mass: the evidence of the spiral rotation curves suggests the existence of large massive haloes around galaxies as the likely habitat of at least some of this missing mass. To probe this region we must study scales inbetween those of single galaxies and great clusters.

3.4 Intermediate scales: Binaries and Small Groups

The familiar problem of finding information about the masses of the components of a spectroscopic binary star system is similar to our problem in studying binary galaxies. However the orbital periods in the latter case are longer than available observing time by far too many orders of magnitude, and the probability of chance superposition of unbound pairs is much higher. THe total mass can only be found up to a projection factor ; this can only be removed by averaging over a statistical sample and careful calculation of the expected average projection factor. Estimates of contamination due to spurious pairs are difficult: FG discuss evidence that there are very few true binaries containing elliptical galaxies. White (1981) discusses analysis of binary galaxy data. He uses data obtained by Turner and by making various different assumptions and choosing different subsamples gets representative values of M/L between 16 and 38.

The problem of contamination is most severe for small groups of galaxies and may cause overestimates of their masses. Values obtained by Turner and Gott, and by Rood and Dickel, from the virial theorem and converted by FG to their standard M/L system indicate M/L =30 to 140 for groups. Rood and Dickel (1978,1979) and Bachall (1981) analyse evidence that M/L increases with velocity dispersion and size of group,implying that the missing mass may be more important for large velocity dispersions.

3.5 Missing Mass nearer home

The rotation curve of the Milky Way has been determined from globular cluster and dwarf satellite galaxy radial velocities. The data has large scatter but the curve seems to flatten out at a value of 220 km/s. Webbink (quoted in FG) studied both radial velocity data and the size of the objects concerned, which is limited by tidal disruption and so gives a measure of the potential. He gets a mass of 1.4×10^{12} $M_{\odot}$ if that mass extends to 200 kpc.

The dynamics of the Local Group can be used to measure its mass. Assuming that whatever form the mass is in, it is almost all associated with M31 and our galaxy, Einasto and Lynden-Bell(1982) calculate the motion of their centre of mass from independent members of the local group and hence derive a mass ratio of the pair. They further deduce a total mass of the two from the orbital motion,which is derived from the velocities of the two galaxies assuming that the periapsis of the orbit was early in the universe. Their results suggest a mass of 1 to 2 $\times 10^{12}$ $M_\odot$ for the Milky Way and about twice that value for M31.

The local dynamical mass may be found by considering the acceleration K_z normal to the galactic plane of stars of a given type. The total mass density is $\rho = \frac{1}{4\pi G}\frac{\partial K_z}{\partial z}$,the 'Oort limit', where the quantity $2(A^2-B^2)$, A and B being Oort's constants,has been neglected relative to the acceleration gradient. The equations of stellar hydrodynamics give

$$K_z = -\frac{1}{n}\frac{\partial}{\partial z}(n\langle v_z{}^2\rangle)$$

where n is the number density of the population of stars being considered. This can in principle be measured to give ρ but it is very difficult to find a suitably well mixed population of stars whose properties are understood (eg whose distances can be reliably estimated.) Jones (1976) finds ρ =0.14 $M_\odot pc^{-3}$. If the luminosity density is 0.05-0.07 $\mathcal{L}_\odot pc^{-3}$ (see later) then M/L_V=2 to 2.8 which is comparable to that found for late type spirals within the Holmberg radius.

3.6 Missing Mass - or cosmic conspiracy ?

As has been emphasized above, the determination of masses and mass-to-light ratios of galaxies and clusters of galaxies remains plagued with uncertainty. Every measurement indicating the presence of dark matter (high mass-to-light ratios) may be explained away as due to one of the many problems discussed in chapter 2. Nevertheless the combination of all the evidence-the flat rotation curves ,and the large dynamical masses on the largest scales-leads me to conclude that most of the mass of the universe is indeed in dark matter, possibly almost entirely in galactic haloes,possibly with some matter spread out between the galaxies. It is still entirely possible that the problem will disappear in ten years time, revealed as a conspiracy of celestial circumstances and invalid assumptions combining to counterfeit the large mass estimates,but at present we must take the problem as it stands and consider the implications. Chapter 4 describes the search for the missing mass.

How can we improve the unsatisfactory observational state of affairs ? Trying to look for something invisible is tricky at best. We need more accurate redshifts and more of them. We need better detectors to seek faint galactic haloes. New instumentation on ground-based telescopes will help. New observatories in space - Exosat,IRAS, Space Telescope - will also help. But these are unlikely to be enough without the help of new theoretical approaches as well. Better understanding of galaxy clustering, and computer simulations of the dynamics of groups of galaxies,may improve our understanding of virial masses (see,e.g.,Gott 1979). Gott (1981) has suggested that the 'gravitational lens' effect may provide a new probe of the outer parts of galaxies, giving the possibility of detecting low mass stars in a halo of a galaxy along the line of sight to a quasar. Such contributory evidence may build up slowly until the existence of the missing mass is established as definitely as the cosmological distances of quasars,so controversial fifteen years ago, have now been confirmed.

4. What is the Missing Mass ?

'We seek him here,we seek him there,
Those Frenchies seek him everywhere.
Is he in heaven? Is he in hell?
That demmed,elusive Pimpernel? '

THE SCARLET PIMPERNEL (1905)

4.1 Mass and Light in the Solar Neighbourhood

Before considering what form the missing mass may be in,we shall investigate the contribution to the mass density from known forms of matter.

The mass-to-light ratio of the Sun-Jupiter system is 1.001 : the amount of mass in other planets,comets etc. in the solar system is believed to be $4x10^{-4}$ $M_{\odot}$ (Allen 1973, hereafter AQ). However, M/L values from other stars range from about 1/5000 (BOI star) to tens of thousands (faint red dwarfs). Hence we must study carefully the distribution of stars of different mass and luminosity.

As a first estimate we count stars within 5 pc of the Sun (after Joeveer and Einasto 1976). This sample is chosen (a) to be reasonably complete: only two new stars are added to the list (and 3 moved within the 5 pc limit) from data published 1969-1978 (Gliese and Jahreiss 1979): (b) to include at least a few early type stars: and (c) to be small enough to be done in a few hours. Stars are taken from AQ,masses are used where iven and otherwise estimated from the mass-luminosity relation tabulated in the same source (which is in agreement with other authors,eg Veeder(1974).) (N.B. I have given Altair the benefit of the doubt- AQ gives 5.07pc but Gliese[22] and Jahreiss give 5.05±0.10).

	Number	Mass ($M_\odot$)	V Luminosity ($\mathcal{L}_\odot$)	M/L_V
Early type stars	3	6	41.5	0.14
G and K stars	12	10	4.9	2
M stars	41	9 ?	0.1	90
White dwarfs	5	3	0.006	500
Total	61	28	46	0.6

It is obvious that almost all the luminosity comes from a small fraction of the mass. In fact one star - Sirius - provides half of the luminosity. It must be emphasized that this calculation is very crude, but it gives a stellar mass density of 0.05 $M_\odot pc^{-3}$ for stars and 0.006 $M_\odot pc^{-3}$ for white dwarfs, which is in agreement with standard values. The calculation also demonstrates that for a reliable M/L value we must sample larger volumes for the rarer, brighter stars. It is also to be noted that M/L_V excluding early type stars is 4.4, so we are not surprised that elliptical galaxies have higher M/L_V.

4.2 The Luminosity Function and the local mass-to-light ratio

The luminosity function $\phi_i(M)$ is the number density of stars of type i (eg giants, main sequence, white dwarfs) with absolute magnitude in a unit interval around M.

The luminosity density is

$$L = \sum_i \int_{-\infty}^{\infty} \mathcal{L}(M)\phi_i(M)\,dM$$

where $\log \mathcal{L}_V(\mathcal{M}) = (4.83 - \mathcal{M})/2.5$

$\log \mathcal{L}_B(\mathcal{M}) = (5.48 - \mathcal{M})/2.5$

The mass density is

$$M = \sum_i \int_{-\infty}^{\infty} M_i[\mathcal{L}(\mathcal{M})]\, \phi_i(\mathcal{M})\, d\mathcal{M}$$

where $M_i(\mathcal{L})$ is the mass-luminosity relation for stars of type i. For faint stars ($M_V>8$) the empirical relation is (Veeder 1974) $M_V=7.0-8.71\log M/M_\odot$ Similar empirical values exist for brighter stars (eg,AQ § 100.) In general stellar masses are unreliable,except for those stars in binaries with suitable orbits.

The luminosity density found from the luminosity function of various investigators (Starikova 1960,McCuskey 1966,Luyten 1968,Wielen 1974) is roughly between 0.05 and 0.07 $\mathcal{L}_\odot\,pc^{-3}$. The light comes almost entirely from stars of absolute magnitude M_V between -2 and +8. The luminosity function quoted in Schwarzschild (1958) suggests that red giants provide one third of this luminosity. The mass density estimated from the luminosity functions is about 0.04 $M_\odot pc^{-3}$. Of this,0.02 $M_\odot pc^{-3}$ is in stars fainter than M_V=+8. (Wielen 1974 has an extra 0.004 $M_\odot pc^{-3}$ in stars of magnitude 12 and 13.) A recent study of the faint end of the luminosity function by Gilmore and Reid (1982) gives a sharper drop at faint magnitudes (13 to 17)- the corresponding mass density is only 0.016 $M_\odot pc^{-3}$ in stars fainter than M_V=+8. Again,bright massive stars are unimportant in the mass density. In all cases extrapolation of the curves to fainter magnitudes suggests there is little extra mass density in faint normal stars. However the faint end of the luminosity function remains uncertain.

To the above mass density must be added that of white dwarfs. I will adodpt the value $0.004M_\odot pc^{-3}$ of Peebles (1980) as representative. From AQ § 119, we find that subdwarfs (0.0016 $M_\odot pc^{-3}$),open clusters ($4\times10^{-5}M_\odot pc^{-3}$),and globular clusters ($10^{-6}M_\odot pc^{-3}$) can be neglected in estimating the total mass density. The energy density of fields (background radiation,starlight,magnetic fields etc) is only 10^{-10} $M_\odot c^2 pc^{-3}$ and may be neglected here and elsewhere. The remaining contribution from known matter is that due to gas and dust.

The density of hydrogen in interstellar space was measured by Savage et al (1977) who found a neutral hydrogen density of 0.021 $M_\odot pc^{-3}$ and an H_2 density of 0.007 $M_\odot pc^{-3}$. Other estimates of molecular hydrogen density have estimated values from the CO density and are less reliable: Solomon and Sanders (1980) suggested that the interstellar medium is actually dominated by giant molecular clouds, and their values give an H_2 density of 0.14 $M_\odot pc^{-3}$ in a ring of clouds 4-8 kpc from the galactic centre, falling to 0.04 $M_\odot pc^{-3}$ at the outer edge of this ring. This value would bring the local mass density up to the Oort limit,but Blitz and Shu (1980) criticize their mass estimates and suggest values comparable with those quoted above.Hence,we will adopt a hydrogen density of 0.028 $M_\odot pc^{-3}$; assuming a hydrogen abundance of 0.75 this gives a gas density of 0.037 $M_\odot pc^{-3}$. Adding in 0.0015 $M_\odot pc^{-3}$ for interstellar dust we find a local interstellar medium density of 0.039 $M_\odot pc^{-3}$.

The total known local mass density is then 0.08 $M_\odot pc^{-3}$ with an accuracy of roughly 0.01 $M_\odot pc^{-3}$, this agrees with Peebles (1980) and FG. The corresponding M/L is 1-2 $M_\odot / L_\odot$, a factor of 2 smaller than the K_z limit. (However,this limit is sufficiently uncertain that we are not forced to accept a significant local missing mass.) It is clear that the missing mass in galaxies and clusters of galaxies is very much darker than normal stellar populations.

4.3 The Missing Mass as 'ordinary' matter

4.3.1 Low mass stars

Perhaps the most obvious possibility is that the mass lies in M dwarfs forming a dark galactic halo. Their M/L_B could be quite high enough to explain the observations. If an early generation of stars (of low metal content) formed while the galaxy was still contracting they might have a quite different mass distribution from that of nearby stars. Dekel et al (1980) studied the formation of a $\rho \propto 1/r^2$ halo by a slow tidal encounter on a $\rho \propto 1/r^3$ halo with an n-body simulation, so the distribution of the stars is not necessarily that at formation. Ostriker and Thuan (1975) presented a model of galactic evolution with a halo presently consisting of low mass stars.

Observations in the infrared to attempt to detect such haloes have been without success. Boughn et al (1981) made infrared observations of NGC4565 to search for halo light on the minor axis of the galaxy (if the missing mass were in a disc, it would not be seen by this method.). They get an upper limit of $M/L_K \geqslant 38$ in the K band assuming a distance of at least 24 Mpc. For comparison the faintest known main sequence star VB10 has $M/L_K=34$ (compare its $M/L_V=36000, M/L_B=160000$): the newly discovered RG0050-2722 is similar. These observations suggest that ordinary stars may be too bright in the infrared to be acceptable candidates for the missing mass.

4.3.2 Black dwarfs and dark remnants

The fact that some nearby stars have invisible, astrometrically discovered companions has led to the suggestion (eg Kumar 1972) that the missing mass lies in 'dark stars'. Some stars are thought to have masses less than $0.07M_\odot$ and some invisible companions seem to be $0.01M_\odot$. As discussed above, very few low mass stars are observed. A search for such stars in binaries with white dwarfs (where they might be easier to detect) by Probst et al. (1982) found no new such stars. Staller and de Jong (1981) suggest that the Space Telescope may be able to detect non-hydrogen burning 'black dwarfs'.

Theoretical studies by Grossmann and Graboske (1971) suggested that the minimum mass for hydrogen burning was about $0.08\ M_\odot$. Lynden-Bell and Low (1976) calculated the minimum Jeans mass for a dark molecular cloud as $0.007M_\odot$, so there is room for a population of stars which would shine only briefly from gravitational energy and then cool. However the formation of galaxies, stars and planets is not sufficiently well understood at present to make confident statements; modern work on the role of magnetic fields in star formation might change the above result.

Another possible source of dark stars is cooled, dead remnants of old (first-generation?) stars. Due to the possible effects of mass loss in their evolution it is not clear what masses would be expected but a substantial contribution is possible. In the immediate solar neighbourhood Joeveer and Einasto (1976) suggested that black dwarfs could contribute about $0.01M_{\odot}pc^{-3}$ and dark remnants a similar amount.

4.3.3 Rocks and comets

Tinsley and Cameron (1974) suggested that the mass of the Sun's cometary cloud could be comparable with the mass of the Sun. In contrast, Clube and Napier (1982) consider comets originating in giant molecular clouds, with large amounts of mass in the planetesimal population. Whatever the merits of these theories, a huge population of planetesimals, if it could be formed, would be practically undetectable and cannot be ruled out. Newman and Cox (1980) consider the possibility that gamma-ray bursts might be caused by such interstellar asteroids hitting neutron stars, but in general rocks lead a quiet life. They also seem quite unlikely to exist as they would presumably be of high density (ie not hydrogen) and would be expected to be associated with a much higher density of gas.

4.3.4 Gas

Observations with orbiting X-ray telescopes have shown that hot gas exists in clusters of galaxies. Lawrence (1978) presented evidence for an extended diffuse source in the Virgo cluster. Forman et al (1979) estimate the Virgo X-ray emitting gas density at 5×10^{-4} cm^{-3}. Cavaliere and Fusco-Fermiano

(1976) estimate the mass of gas in the Coma cluster,getting 20% of the virial mass. Much better data is now available. Work by Fabian and Nulsen (in preparation) on the cluster 0340-538 confirms that gas provides only of order 10% the virial mass. Gas densities in rich clusters can vary from less than 5×10^{-3} cm^{-3} to 0.1 cm^{-3}.(Fabian and Kembhavi 1982)

On larger scales,it has been suggested that the density of the intergalactic medium might be cosmologically significant. The isotropic Xray background fits thermal bremmstrahlung quite well between 3 and 50 keV and could be due to intergalactic gas. At least some is due to unresolved sources and it is possible that all of it could be due to faint quasars and the contribution from Seyferts.Fabian and Kembhavi (1982) fit the spectrum to a hot gas expanding and cooling with the universe,taking off the Seyfert contribution but not the quasar one. Their typical fit and representative result gives $\Omega_{gas} \simeq 0.04$ which is not enough to give the observed dynamics on the largest scales.

4.3.5 Black Holes

The mass in galactic haloes and clusters could come from massive black holes. Black hole remnants of a first generation of galactic stars would have to be more massive than $100M_\odot$ to avoid producing too much light in their stellar phase. If the first generation of stars was very massive this scenario might work. Alternatively a first generation of stars might have formed before galaxies (but after decoupling) and left black hole remnants. The limits on background light then allow even holes of less than $100M_\odot$. Lumps of 10^6 $M_\odot$ might collapse directly to become holes. Further,inhomogeneities in the early universe could produce black holes: those formed after nucleosynthesis would have masses over $10^7M_\odot$ and would be unlikely to contribute significantly to the density. (Carr 1978,1980)

Nonstandard cosmological models may produce many black holes. Carr(1977) discusses galaxy formation in a cold big bang with haloes of $10^6M_\odot$ black holes.

4.4 'Exotic' explanations for the Missing Mass

' Why, sometimes I've believed as many as six

impossible things before breakfast. '

The Red Queen, in 'THROUGH THE LOOKING GLASS'.

4.4.1 Massive neutrinos

Recent developments in theory and experiment have renewed interest in the possibility that neutrinos may have a mass. At present there is no widely accepted experimental evidence for this, but it has been suggested that the missing mass might be in τ neutrinos of about 30 eV even if electron neutrinos have a mass too small to measure. (Sciama 1982) A neutrino mass of 24 eV (100 eV if $H_0=100$) would close spacetime: 3 eV would dominate the mass of the universe, if $\Omega_{baryons}=0.1$. Relic neutrinos left over from the Big Bang will have a velocity of about 9 ($20eV/m_\nu$) km/s. Tremaine and Gunn (1979) point out that the exclusion principle and phase space considerations limit the number of neutrinos that can be stuffed into galactic haloes, but their conclusion that neutrinos cannot provide the missing mass is disputed by later authors. Schramm and Steigman (1981) suggest that relic neutrinos of mass 4-20eV would cluster on the scales of clusters and binary galaxies, and for slightly larger masses would contribute to the mass in galactic haloes too. There are problems with galaxy formation, though, as massive neutrinos may erase primordial perturbations on scales smaller than clusters, and if galaxies then collapsed from these clusters, neutrinos would be too hot to cluster with them. Much work is now being done on trying to prove or disprove the possibility of neutrino dominated galaxy formation. (eg. Davis et al 1981, and a number of papers in the recent 10th Texas symposium.)

4.4.2 Primordial black holes

Hawking(1971) suggested that there might be a large number of small black holes created by anisotropy in the early universe. Inhomogeneities in a conventional model can also produce such holes which can form if they have of order the particle horizon mass: thus holes forming at time t would have masses $10^5(t/1sec)\ M_\odot$. Black holes of less than 10^{15}g would have exploded by now, and a limit on the mini-black hole density is given by the gamma-ray background. (Carr 1978). It is however possible that primordial black holes of masses between 10^{-16} and $10^5\ M_\odot$ contribute significant density. If the early universe equation of state was 'stiff' the primordial black holes might have masses starting at 10 $M_\odot$. (Carr 1980)

4.4.3 Gravitational Waves

Rees(1971) suggested that very long wavelength (1-10 Mpc) gravitational waves created in the big bang could increase the velocity dispersion in clusters and groups and so explain the missing mass problem there. (Gravitational waves can't fix galactic rotation curves.) However Jackson (1972) criticizes this theory, claiming that it does not adequately explain cluster galaxies' distribution in redshift space. Carr (1980) derives a limit of $\Omega_{GW}=10^{-4}$ from nucleosynthesis considerations,and much stronger limits for the long wavelength waves considered here. So, it appears that this is not a likely suspect for the missing mass.

4.4.4 <u>A trip to the (high-energy) zoo</u>

Many particles exist - at least in the imagination of theorists - which can be considered as possible candidates for the missing mass. 'Photinos' and similar particles predicted by supersymmetry could take the place of massive neutrinos if the latter do not exist. Particles such as right-handed neutrinos,(Klinkhamer et al 1981) ,Higgs mesons and axions would decay too quickly. If magnetic monopoles exist,they could not provide the missing mass as they would destroy galactic magnetic fields. (Dolgov and Zeldovich 1981).

4.4.5 <u>Missing Mass- or missing physics ?</u>

The evidence for invisible mass in groups and clusters of galaxies depends crucially on the validity of Newtonian gravitation,as the appropriate limiting case of general relativity (GR). While standard bread-and-butter physics appears to be true on laboratory scales (Held and Yodzis 1981), graviation theory has not been tested at the scale of clusters of galaxies (except insofar as GR explains cosmological redshifts) and it is possible that the law of gravitation -and hence the virial theorem - may require modification for large masses. (Ginzburg 1975). Jackson (1970) suggests that a negative cosmological constant could explain the missing mass problem within the framework of GR,but Rood and Dickel (1979) conclude from their analysis of virial properties of groups that a cosmological constant would not produce the observed correlations. Klinkhamer(1980) derives the virial theorem in the scale covariant graviation theory of Canuto and colleagues.He claims that virial masses are reduced by a factor of 2 due to the addition of an extra term in the equations. However with no independent evidence for nonstandard gravitation this approach is something of a last resort.

5. Conclusion and 'Best Buys'

' As I was going up the stair
I met a man who wasn't there.
He wasn't there again today,
I wish,I wish he'd stay away. '

('The Psychoed',by H.Mearns)

Many astronomers indeed wish the problem of the missing mass would ' stay away', but it shows no sign of doing so. How will the problem be resolved? It may be interesting to recall (as pointed out by Rood and Dickel 1979) that this is not the first 'missing mass' problem in astronomy. The celestial mechanics of the solar system ran into a similarly embarrassing difficulty beginning in the late 18^{th} century when,again, the law of gravitation gave the wrong answers. This problem,as Cambridge astronomers well know, was resolved in 1846 when the perturbing mass of Neptune was identified and discovered. However, a similar problem later, with the orbit of Mercury, turned out to be due to missing physics,not missing mass. Thus history provides no clear guide,but cautions us to keep an open mind.

Low mass stars are perhaps the least exotic (some would say most boring) explanation of the missing mass,if it is real. However -or perhaps therefore- they seem the most likely of the candidates still consistent with observation to have been produced,with black hole remnants next, either choice probably forming in an early generation of stars. However,if infrared measurements rule out the former,and if nucleosynthesis arguments remain in conflict with cluster mass-to-light ratios,more radical explanations will be required. In that case , massive neutrinos or other, fancier,elementary particles seem the likeliest candidates at the moment.

Further developments in instrumentation,observation,and theoretical understanding of star formation and death,galaxy formation,analysis of velocity dispersions, cosmology and particle physics will be required - and are likely to occur- over the next decade to help establish finally the existence and nature of the missing mass - in other words, to find out what the Universe is made of.

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